Competitive Search Equilibrium with Asymmetric Information*

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1 Introduction

This paper analyzes the implications of asymmetric information in a labor market with search frictions. A worker and firm agree upon an employment contract in an environment with symmetric information. Afterwards, the firm observes the worker's match-specific productivity x, but the worker continues to know only the underlying productivity distribution $F(x)$. Depending on the incentives generated by the employment contract and any information that the firm chooses to reveal to the worker, the employment contract then dictates the probability that the worker is employed and the wage that she receives if she is employed. If the worker is employed, she produces x, while otherwise her productivity is normalized to zero.

This information structure fits naturally into a competitive search model (Moen 1997, Shimer 1996, Mortensen and Wright 2002). I consider the interaction between a large number of workers and a large number of firms in a static economy with search frictions. Firms compete for workers by advertising an employment contract, which in general says "if you contact me and I tell you that your productivity is x, I will hire you with probability $e(x)$ and transfer you $t(x)$ regardless of whether I employ you." Each worker looks at the menu of

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available employment contracts, computes the probability that she will succeed in contacting a firm offering that contract, evaluates the reporting strategy that a firm offering a particular contract will use, and decides where to apply for a job. In particular, workers recognize that if the ratio of firms offering some contract to workers seeking that contract θ is very high, the probability of contacting a such a firm $\mu(\theta)$ will be higher. From the firm's perspective, it is easier to contact a worker when the ratio of firms offering to workers seeking a contract is low, so $\mu(\theta)/\theta$ is a decreasing function.

Alternatively, following Mortensen and Wright (2002), this can be reinterpreted as an environment with a competitive sector of 'market makers'. A market maker announces that if a firm wants to enter his market, it must commit to a particular employment contract and pay the market maker a nonnegative entry fee; and if a worker wants to enter his market, the worker must agree to accept the same employment contract and pay the market maker a nonnegative entry fee. Within the market, there is random matching, so if θ is the firm-worker ratio, a worker contacts a firm with probability $\mu(\theta)$ and a firm contacts a worker with probability $\mu(\theta)/\theta$. Market makers compete by choosing the 'best' possible employment contract and by lowering the required entry fees. In equilibrium, competition drives the entry fees, and hence the market makers' profit, to zero, while market makers select the same contract that firms choose in the contracting-posting game described above. A third isomorphic possibility is that workers select and advertise contracts in an effort to attract potential employers.

In all three environments, there is a critical contracting problem: how is an employment contract best structured so as to exploit all the possible gains from trade given the constraints imposed by asymmetric information? If there are no restrictions on the menu of feasible contracts, the answer is simple. A firm must transfer t to any worker it contacts and employ the worker if her productivity exceeds her outside option of zero. Such a contract is incentive compatible and ensures that the firm employs the worker whenever doing so is bilaterally efficient. Moreover, by varying the transfer t, any division of the surplus generated by a match is feasible.

To make the asymmetric information problem interesting, I introduce an additional restriction on employment contracts: a firm always has the option of announcing that the worker's productivity is zero, in which event the contract specifies a zero transfer, $t(0) = 0$. I interpret this as an adverse selection problem. More precisely, there is a small number of unproductive workers who always produce 0. If $t(0)$ were positive in a single sub-market and zero in all others, that market would attract all the unproductive workers, making it unattractive both for firms (who would be less likely to find a productive agent) and workers (who would be less likely to find a firm). Therefore competition drives $t(0)$ to zero in all open markets.

This single restriction has a profound effect on the nature of optimal contracts. Under a plausible condition on the productivity distribution F,[1] the best contract is perhaps the simplest mechanism that one could imagine: the firm promises to hire the worker and pay her w if $x \geq w$ and otherwise promises not to hire the worker and to pay her zero. Equivalently, one can think of this as a simple fixed wage contract, where the firm sets the wage w before observing the workers' productivity and then has the right to determine whether to employ the worker.

More generally, the best contract can have a slightly more complicated characterization: the contract specifies two wages $w_1 < w_2$ and a probability $\pi \in (0,1)$. When a worker and firm meet, the firm randomly (but publicly) selects a wage, choosing w_1 with probability π and w_2 otherwise. It then observes both the wage and the worker's productivity and decides whether to hire her at the selected wage or not to hire her and pay her nothing. There is no other contract, no matter how complex, that does better in a competitive search equilibrium with asymmetric information.

To my knowledge, only Faig and Jerez (2004) have previously examined a competitive search model with asymmetric information. That paper is much more ambitious than this one, in that the authors build a quantifiable theory of commerce, while I focus on the simplest possible model of search and asymmetric information. This allows me to generalize Faig and Jerez's (2004) findings along some dimensions. In particular, those authors assume that the distribution of productivity F takes a particular functional form (uniform on $[0,1]$), while I allow for a general distribution, and those authors restrict attention to mechanisms that use pure strategies, while I allow for public randomization. The latter assumption is critical for my results, since if public randomization were impermissible, i.e. the employment probability $e(x)$ were restricted to be either zero or one, it is easy to show that any incentive compatible contract would simply impose employment above some threshold w at a wage of w.

Most previous authors who have examined asymmetric information in search models have presumed that wages are determined by a particular bargaining procedure. For example, Trejos (1999) examines a monetary model of exchange with asymmetric information. The structure of the bargaining game follows Rubinstein (1982): nature randomly selects one party to make an offer to the other; if the offer is rejected, there is a short delay before the

[1] The condition is that $\frac{xF'(x)}{1-F(x)}$ is nondecreasing.

random selection procedure is repeated. Trejos (1999) shows that the equilibrium of this game is equivalent to the axiomatic Nash bargaining solution in a model with symmetric information. In particular, all the bilateral gains from trade are exploited. Berentsen and Rocheteau (2003) extend Trejos by allowing for divisible money and divisible goods; however, they maintain his focus on the axiomatic Nash bargaining solution of the related symmetric information model.

The labor economics literature has focused on inefficiencies that may arise in the presence of asymmetric information. Acemoglu (1995) assumes that (uninformed) workers make wage offers to (informed) firms. If an offer is rejected, the worker may make another offer in the following period. This introduces bilaterally inefficient delay in equilibrium, with a worker initially demanding a high wage before gradually reducing her wage demand upon deducing that she is in a low productivity matches. Kennan (2004) and Tawara (2004) allow nature to randomly select one of the parties to make a take-it-or-leave-it offer to the other party. If the worker gets to make the offer, she behaves as in Acemoglu (1995), while if the firm makes the offer, it pays the worker her reservation wage assuming that output exceeds this low threshold. This superficially resembles the optimal contract, since the worker is always employed at sufficiently high productivity realizations, is sometimes employed at intermediate productivity realizations (when the firm makes the offer), and is never employed at low productivity realizations, below the workers's reservation wage.[2] But despite this, the two thresholds are never optimal. For example, an optimal contract would dictate that a worker producing slightly more than her reservation wage should never be employed, while the particular bargaining game implies that the worker would be employed if the firm makes the wage offer. This implies that a worker and firm could obtain a Pareto improvement by agreeing to an employment contract of the sort described here before the firm observes the match specific productivity realization.

Curtis and Wright (2003) obtain similar results in a model that superficially looks quite different. They consider a dynamic model in which buyers and sellers meet sequentially. When they meet, the seller sets a price and the buyer privately observes her idiosyncratic valuation for the seller's good. Trade occurs if the buyer's valuation exceeds the seller's price. The authors show that there are generically at most two prices in equilibrium, and under the same regularity condition as the one developed in this paper, the equilibrium price is unique. If there are multiple prices, then trade occurs with probability one if the

[2] Kennan (2004) focuses on parameter values such that all meetings result in matches. Tawara (2004) considers this more general case.

buyer realizes a sufficiently high valuation, with an intermediate probability when the buyer realizes a valuation between the two prices, and with zero probability at a low valuation, similar to the employment probability function in this paper. But there are three notable differences between the two papers. First, my analysis works in a static model, while Curtis and Wright (2003) require a dynamic framework in order to have a two price equilibrium. Second, I allow sellers (firms) to offer general mechanisms, while Curtis and Wright restrict firms to offer a single price. Finally, in a competitive search model, each firm must randomly select a price, while in Curtis and Wright's (2003) random search model, it is enough that a fraction of firms offer a low price and the remaining firms offer a high price.

I proceed through the analysis in three stages. Section 2 analyzes a static environment with just three possible productivity realizations, $0 < x_1 < x_2$. This simple environment facilitates demonstrating the main forces at work. Section 3 extends this to a continuum of productivity levels and derives the main characterization of a competitive search equilibrium with asymmetric information. Section 4 explains why the main results go through in a dynamic environment.

2 Three Productivity Realizations

2.1 Setup

There are a large number of ex ante identical risk-neutral workers and a large number of risk-neutral firms. When a worker and firm meet, the pair realize a match-specific productivity level taking values $0 \equiv x_0 < x_1 < x_2$ with probabilities p_0, p_1, and p_2, respectively. The firm observes x_i and the worker does not, although there is common knowledge about the economic environment.

At the start of the period, firms can create vacancies at cost c. Each vacancy entitles a firm to post an employment contract. For standard reasons, I restrict attention to incentive compatible contracts: if a worker contacts the vacancy, the firm announces the productivity level x_i, $i \in \{0, 1, 2\}$, makes a payment $t_i \geq 0$, and hires the worker with probability $e_i \in [0, 1]$. Incentive compatibility means that a firm is willing to truthfully reveal its productivity realization,

$$e_i x_i - t_i \geq e_j x_i - t_j$$

for all i and j. In addition, I impose that $e_0 = t_0 = 0$, so a worker who produces nothing is not hired and is paid nothing. There are two possible justifications for this. The first is an ad hoc assumption that a firm cannot commit to pay a worker who does not produce any output, perhaps due to a liquidity constraint. Alternatively, Section 2.3 justifies this as an equilibrium condition in the presence of a small amount of adverse selection. Note that when $e_j > 0$, we may reinterpret the transfer as a wage payment $w_j = t_j/e_j$, with the worker getting zero income when the match is not consummated.

After firms commit to wage contracts, workers observe all the contracts and decide where to apply for a job. Workers and firms anticipate that each contract is associated with a vacancy-unemployment (v-u) ratio θ, and that any worker seeking a firm offering that contract finds one with probability $\mu(\theta)$ while any firm offering the contract finds a worker with probability $\mu(\theta)/\theta$. In this section, I assume $\mu(\theta) = \mu_0\theta^{1-\alpha}$, $\alpha \in (0,1)$ with $\mu_0 > 0$ sufficiently small so that $\mu(\theta) < \min\{1, \theta\}$. The v-u ratio adjusts so that workers are indifferent about which contract to seek; let V denote the expected utility for an unemployed worker, which is constant across contracts.

The competitive search equilibrium can be represented as a tuple $\{V, \theta, t_1, t_2, e_1, e_2\}$ that solves the following constrained optimization problem:

$$
\begin{aligned}
c = \max_{\theta,t_1,t_2,e_1,e_2} \ & \frac{\mu(\theta)}{\theta}\Big(p_1\big(e_1x_1 - t_1\big) + p_2\big(e_2x_2 - t_2\big)\Big) \\
\text{subject to } & \mu(\theta)\big(p_1t_1 + p_2t_2\big) \geq V \\
& e_2x_2 - t_2 \geq e_1x_2 - t_1 \\
& e_1x_1 - t_1 \geq e_2x_1 - t_2 \\
& e_2x_2 - t_2 \geq 0 \\
& e_1x_1 - t_1 \geq 0 \\
& e_i \in [0,1]
\end{aligned}
$$

A firm chooses incentive compatible transfers t_1 and t_2 and employment probabilities e_1 and e_2 in order to maximize its expected profits, taking as given that the v-u ratio θ will adjust so that a worker seeking this contract receives utility V. Moreover, the maximized level of profit must equal the sunk cost of a vacancy c. It is mathematically easier to analyze the dual problem of a worker choosing $\{\theta, t_1, t_2, e_1, e_2\}$ in order to maximize her utility, taking as given that firms earn zero profits and the four incentive compatibility constraints.

2.2 Characterization

As one might expect, firms always employ workers when a good match is realized:

Lemma 1. *$e_2 = 1$.*

Proof. Suppose the solution to this problem is a tuple $\{\theta, t_1, t_2, e_1, e_2\}$ with $e_2 < 1$. I will find another tuple $\{\theta', t_1, t_2, e_1', e_2'\}$ that is both feasible and delivers a higher value of the objective function.

First, sum the first and second IC constraints to get $(e_2 - e_1)(x_2 - x_1) \geq 0$, so $e_2 \geq e_1$. This implies it is feasible to increase both e_1 and e_2 by equal amounts, say to $e_1' = e_1 - e_2 + 1$ and $e_2' = 1$. All four IC constraints continue to hold with these new employment probabilities if the transfers t_1 and t_2 are left unchanged. Next, the new value of θ is determined from the free entry condition. Since $p_1 e_1' x_1 + p_2 e_2' x_2$ is larger than $p_1 e_1 x_1 + p_2 e_2 x_2$ and $\mu(\theta)/\theta$ is a decreasing function, θ' is also higher than θ. This increases the value of the objective function. $\square$

This simplifies the problem slightly. I can express the dual problem as

$$V = \max_{\theta, t_1, t_2, e_1} \mu(\theta)\big(p_1 t_1 + p_2 t_2\big) \tag{1}$$

$$\text{subject to } c = \frac{\mu(\theta)}{\theta}\Big(p_1\big(e_1 x_1 - t_1\big) + p_2\big(x_2 - t_2\big)\Big) \tag{2}$$

$$x_2 - t_2 \geq e_1 x_2 - t_1 \tag{IC–1}$$

$$e_1 x_1 - t_1 \geq x_1 - t_2 \tag{IC–2}$$

$$x_2 - t_2 \geq 0 \tag{IC–3}$$

$$e_1 x_1 - t_1 \geq 0 \tag{IC–4}$$

$$e_1 \in [0, 1] \tag{3}$$

I solve this problem by characterizing the solution in different regions of the parameter space. I start with the case when x_1 and x_2 are sufficiently similar so that the incentive constraints do not affect the solution to the optimization problem.[3]

[3]It is incorrect to say that the incentive constraints do not bind. In the absence of an asymmetric information problem, only the average transfer $p_1 t_1 + p_2 t_2$ is determined in equilibrium. Asymmetric information ensures $t_1 = t_2$ since the first two incentive constraints bind. Nevertheless, the incentive constraints do not affect workers' expected utility V, the v-u ratio θ, or the employment probability e_1.

Proposition 1. *Suppose* $x_2 > x_1 \geq \frac{\alpha p_2 x_2}{p_1(1-\alpha)+p_2}$. *Then* $e_1 = 1$,

$$t_1 = t_2 = \frac{\alpha(p_1 x_1 + p_2 x_2)}{p_1 + p_2} \leq x_1,$$

and θ *solves*

$$(1-\alpha)\mu_0 \theta^{-\alpha}\big(p_1 x_1 + p_2 x_2\big) = c.$$

Proof. Consider a modified version of the optimization problem without any incentive constraints:

$$\begin{aligned} V = &\max_{\theta, t_1, t_2, e_1} \mu(\theta)\big(p_1 t_1 + p_2 t_2\big) \\ \text{subject to } c = &\frac{\mu(\theta)}{\theta}\Big(p_1\big(e_1 x_1 - t_1\big) + p_2\big(x_2 - t_2\big)\Big) \\ &e_1 \in [0, 1] \end{aligned}$$

Use the zero profit condition to eliminate $\mu(\theta)(p_1 t_1 + p_2 t_2)$ from the objective function:

$$V = \max_{\theta, e_1} \mu(\theta)(p_1 e_1 x_1 + p_2 x_2) - c\theta \text{ subject to } e_1 \in [0, 1].$$

The objective function is increasing in the employment probability e_1, and so it is optimal to set $e_1 = 1$. Then θ satisfies the (necessary and sufficient) first order condition $\mu'(\theta)(p_1 x_1 + p_2 x_2) = c$. With the assumed functional form for μ, this reduces to the desired equation for θ. Next, use the zero profit condition to compute

$$p_1 t_1 + p_2 t_2 = p_1 x_1 + p_2 x_2 - c\frac{\theta}{\mu(\theta)} = \left(1 - \frac{\theta \mu'(\theta)}{\mu(\theta)}\right)(p_1 x_1 + p_2 x_2),$$

where the second equality uses the first order condition for θ to eliminate c. Note that under the functional form assumption for μ, its elasticity is simply $1-\alpha$, so $p_1 t_1 + p_2 t_2 = \alpha(p_1 x_1 + p_2 x_2)$.

Now examine the incentive constraints (IC–1)–(IC–4). Since $e_1 = 1$, constraints (IC–1) and (IC–2) imply $t_1 = t_2 = t$. Using the previous equation, this implies

$$t = \frac{\alpha(p_1 x_1 + p_2 x_2)}{p_1 + p_2}.$$

Constraints (IC–3) and (IC–4) require $x_2 \geq t$ and $x_1 \geq t$, respectively. Since $x_2 \geq x_1$, the proposed tuple solves the original constrained optimization problem if and only if $t \le x_1$ or equivalently $x_1 \ge \frac{\alpha p_2 x_2}{p_1(1-\alpha)+p_2}$. $\square$

Next turn to the opposite case, when x_1 is much smaller than x_2. In this case, the probability that a bad meeting results in a match, e_1, is less than one. This helps to reduce the incentive for firms to report a bad meeting when the true match quality is good. Moreover, since the output in a bad meeting is relatively small, the cost of eliminating some bad matches is also relatively small.

Proposition 2. *Suppose* $\frac{\alpha p_2 x_2}{p_1+p_2} > x_1 \ge 0$. *Then*

$$e_1 = \frac{(1-\alpha)p_2 x_2}{p_2 x_2 - (p_1+p_2)x_1},$$

$t_1 = e_1 x_1$, $t_2 = x_2 - e_1(x_2 - x_1)$, *and* θ *solves*

$$c = \frac{(1-\alpha)\mu_0 \theta^{-\alpha} p_2^2 x_2 (x_2 - x_1)}{p_2 x_2 - (p_1+p_2)x_1}.$$

Proof. I characterize the equilibrium when $e_1 < 1$. If incentive constraint (IC–1) were slack, it would be possible to raise e_1 without affecting any of the constraints (IC–1)–(IC–4) or (3). Then the zero profit condition (2) would permit an increase in θ, raising the value of the program, a contradiction. Therefore (IC–1) must bind. Next turn to (IC–2). If it binds, then it and (IC–1) together imply $(1-e_1)x_2 = t_2 - t_1 = (1-e_1)x_1$ or $x_1 = x_2$, a contradiction. So (IC–2) is slack.

Third, look at (IC–4). If it were slack, it would be possible to raise t_1 and reduce t_2, leaving the expected transfer $p_1 t_1 + p_2 t_2$ unchanged. This would relax constraints (IC–1) and (IC–3), tightening only the non-binding constraint (IC–2). But with (IC–1) slack, the earlier argument implies it would be possible to raise e_1 and increase the value of the program, a contradiction. Therefore, (IC–4) must bind. Finally, if (IC–3) binds then it, (IC–1), and (IC–4) imply $e_1 x_2 - t_1 = 0 = e_1 x_1 - t_1$, which again implies $x_1 = x_2$, a contradiction.

In summary, when $e_1 < 1$, the optimization problem is

$$
\begin{aligned}
V = \max_{\theta, t_1, t_2, e_1} & \ \mu(\theta)\big(p_1 t_1 + p_2 t_2\big) \\
\text{subject to } c = & \ \frac{\mu(\theta)}{\theta}\Big(p_1\big(e_1 x_1 - t_1\big) + p_2\big(x_2 - t_2\big)\Big) \\
& \ x_2 - t_2 = e_1 x_2 - t_1 \\
& \ e_1 x_1 = t_1 \\
& \ e_1 \in [0,1],
\end{aligned}
$$

with the remaining constraints slack. Use the constraints to eliminate t_1, t_2, and e_1, temporarily suppressing the requirement that $e_1 \in [0,1]$:

$$
V = \max_{\theta} \mu(\theta) p_2 x_2 - \frac{\big(p_2 x_2 - (p_1 + p_2) x_1\big)\theta c}{p_2(x_2 - x_1)}.
$$

The first order condition is

$$
\mu'(\theta) p_2 x_2 = \frac{\big(p_2 x_2 - (p_1 + p_2) x_1\big) c}{p_2(x_2 - x_1)}.
$$

Substitute this into the constraints to show

$$
e_1 = \frac{(1-\alpha) p_2 x_2}{p_2 x_2 - (p_1 + p_2) x_1}.
$$

The characterization of t_1 and t_2 follows from the two binding incentive constraints (IC–1) and (IC–4).

For this solution to be sensible, we require that $e_1 \in [0,1]$ or

$$
x_1 \leq \frac{\alpha p_2 x_2}{p_1 + p_2}.
$$

Note that when $x_1 = 0$, $e_1 = 1 - \alpha$. It is clear that any lower value of e_1 (such as $e_1 = 0$) would yield the same value of the program in this extreme case. □

Finally I turn to the intermediate case, when x_1 is too small to permit an unconstrained optimum but sufficiently large such that all bad matches are consummated. In this case, firms choose to reduce wages compared to the symmetric information benchmark, which ensures truthful revelation.

Proposition 3. *Suppose* $\frac{\alpha p_2 x_2}{p_1(1-\alpha)+p_2} > x_1 \geq \frac{\alpha p_2 x_2}{p_1+p_2}$. *Then* $e_1 = 1$, $t_1 = t_2 = x_1$, *and* θ *solves*

$$c = \mu_0 \theta^{-\alpha} p_2(x_2 - x_1).$$

Proof. The proof of Proposition 2 implies that $e_1 = 1$. Then constraints (IC–1) and (IC–2) imply $t_1 = t_2$. A comparison of constraints (IC–3) and (IC–4) shows that the former is slack, while constraint (IC–4) must bind, for otherwise Proposition 1 would apply. This implies $t_1 = t_2 = x_1$. Finally, the zero profit condition (2) pins down the value of θ. $\square$

One can verify that when $x_1 = \frac{\alpha p_2 x_2}{p_1(1-\alpha)+p_2}$, Propositions 1 and 3 imply the same values for the four choice variables, while when $x_1 = \frac{\alpha p_2 x_2}{p_1+p_2}$, Propositions 2 and 3 coincide.

2.3 Adverse Selection

Formally, I assume that there are two types of workers, good and bad. A fraction $1 - \varepsilon > p_1 + p_2$ of workers are good and ε are bad. When a good worker contacts a firm, the realized productivity is x_1 with probability $\frac{p_1}{1-\varepsilon}$ and x_2 with probability $\frac{p_2}{1-\varepsilon}$; otherwise it is $x_0 = 0$. When a bad worker contacts a firm, the realized productivity is always equal to 0. A worker knows her type but a firm can only observed the realized productivity. Note that from a firm's perspective, the probability productivity is x_1 or x_2 is just p_1 or p_2.

In this environment, suppose that in fact all other firms offered a zero payment to a worker who produces zero output. Then any deviating firm which set $t_0 > 0$ would attract a positive measure of the bad workers (so $\theta = 0$) and no good workers, making such a policy unprofitable. Thus it is an equilibrium for all firms to set $t_0 = 0$.

Note that without the constraint that $t_0 = 0$ (and ignoring the adverse selection problem), asymmetric information would not be a problem. A firm would simply set $t_0 = t_1 = t_2$ and $e_1 = e_2 = 1$, with transfers chosen at the appropriate level so as to get the (symmetric information) optimal v-u ratio. It is straightforward to show that all the IC constraints are satisfied in this case.

2.4 Summary

To summarize, consider the following numerical example: $p_1 = p_2 < \frac{1}{2}$ and $\alpha = \frac{1}{2}$. Then there are three regions in the parameter space. In region I, $1 > \frac{x_1}{x_2} \geq \frac{1}{3}$, the firm is unconstrained by information problems and chooses the v-u ratio optimally. In region II, $\frac{1}{3} > \frac{x_1}{x_2} \geq \frac{1}{4}$, the firm is constrained by information problems so $t_1 = t_2 = x_1$, but still promises to employ a worker in a low productivity match with probability 1. In region III, $\frac{1}{4} > \frac{x_1}{x_2} \geq 0$, the firm uses the public randomization device in low productivity matches and pays a higher wage to workers in high productivity matches.

3 Continuum of Productivity Realizations

This section generalizes the previous model in two directions. First, I assume that the matching function $\mu(\theta)$ is strictly concave and satisfies $\mu(\theta) < \min\{1, \theta\}$ and in particular $\mu(0) = 0$. This generalizes the Cobb-Douglas matching function from the previous section. Second, and more importantly, I assume that the realized level of productivity x is a random variable with cumulative distribution $F(x)$ and convex support $[0, \bar{x}]$, where $\bar{x} \leq \infty$. I assume $F(\bar{x}) = 1$ but allow $F(0) > 0$, in which case I assume there is a positive probability that a match is unproductive. Finally, it is convenient for the exposition of the results to assume that $\mu(\theta)$ is continuously differentiable with

$$\mu'(0) > \frac{c}{\int_0^{\bar{x}} x dF(x)} > \lim_{\theta \to \infty} \mu'(\theta).$$

One might expect that with a continuum of different productivity levels, an optimal contract would potentially involve a continuum of different wages and employment probabilities. I will show that, perhaps surprisingly, the characterization of equilibrium is, if anything, simpler in this environment.

I represent a competitive search equilibrium with asymmetric information as a constrained optimization problem. I focus on the dual problem, in which the worker chooses the v-u ratio θ, transfer $t(x)$, and employment probabilities $e(x)$ in each type of match:

$$\begin{aligned} V = \max_{\theta, t, e} \; & \mu(\theta) \int_0^{\bar{x}} t(x) dF(x) \\ \text{subject to } c = & \frac{\mu(\theta)}{\theta} \int_0^{\bar{x}} \big(x e(x) - t(x) \big) dF(x) \\ & x e(x) - t(x) \geq x e(y) - t(y) \text{ for all } \{x, y\} \in [0, \bar{x}]^2 \\ & t(0) = 0 \\ & e(x) \in [0, 1] \text{ for all } x \in [0, \bar{x}]. \end{aligned}$$

The worker attempts to maximize her expected utility, but must ensure that the firm earns zero expected profits, the firm truthfully reveals the productivity, and the transfer to unproductive workers is zero. The last assumption again represents an unmodelled moral hazard problem.

3.1 Symmetric Information Benchmark

Before characterizing the equilibrium with asymmetric information, it is worth stepping back to describe a competitive search equilibrium with symmetric information, i.e. the solution to the constrained optimization problem without the set of incentive constraints. Eliminate $t(x)$ from the objective function using the zero profit constraint to get

$$V = \max_{\theta,e} \mu(\theta) \int_0^{\bar{x}} xe(x)dF(x) - c\theta$$
$$e(x) \in [0,1] \text{ for all } x \in [0,\bar{x}].$$

The solution is clearly to set $e(x) = 1$ for all $x > 0$, and then choose a v-u ratio that satisfies the necessary and sufficient first order condition

$$\mu'(\theta) \int_0^{\bar{x}} xdF(x) = c. \tag{4}$$

Given the assumptions on μ, this defines a unique v-u ratio $\theta^s \in (0,\infty)$.

It is straightforward to prove that such an allocation is not incentive compatible. If $e(x) = 1$ for all x, the incentive constraint $xe(x) - t(x) \geq xe(y) - t(y)$ implies $t(x)$ is constant for all x. Since $t(0) = 0$, $t(x) = 0$ as well. Then the free entry condition implies $\frac{\mu(\theta^s)}{\theta^s} \int_0^{\bar{x}} xdF(x) = c$, which is consistent with the first order condition (4) if and only if $\mu(\theta^s) = \theta^s \mu'(\theta^s)$. But strict concavity of μ precludes this possibility at any positive value of θ^s.

3.2 Characterization of Incentive Compatibility

One instead must characterize the competitive search equilibrium with asymmetric information directly. The problem appears complicated, since there is a two-dimensional continuum of incentive constraints. Fortunately, it is possible to simplify the problem considerably by providing a simple characterization of incentive compatible transfer schemes using something like a 'first order approach' (Mirrlees 1971, Laffont and Maskin 1980, Rogerson 1985, Milgrom and Segal 2002):

Lemma 2. *The following two conditions are identical:*

$$A: \quad xe(x) - t(x) \geq xe(y) - t(y) \text{ for all } \{x, y\} \in [0, \bar{x}]^2 \text{ and } t(0) = 0$$

$$B: \quad t(x) = xe(x) - \int_0^x e(y)dy \text{ and } e(x) \text{ is a nondecreasing function.}$$

Proof. I first prove that condition A implies condition B. If $xe(x) - t(x) \geq xe(y) - t(y)$ for all x and y, then this true in particular for y close to x. This implies the generalized first order condition $dt(y) = yde(y)$. Integrate this condition using the boundary condition $t(0) = 0$ to get

$$\int_0^x dt(y) = \int_0^x yde(y) \Rightarrow t(x) = xe(x) - \int_0^x e(y)dy.$$

This is half of condition B. To prove monotonicity of the employment probability, sum the incentive compatibility constraints $\{x, y\}$ and $\{y, x\}$:

$$xe(x) - t(x) + ye(y) - t(y) \geq xe(y) - t(y) + ye(x) - t(x) \Rightarrow \big(e(x) - e(y)\big)(x - y) \geq 0.$$

This proves $e(x) \geq e(y)$ when $x > y$.

Now I prove that condition B implies condition A. Under the proposed transfer scheme, for any $x < y$,

$$xe(x) - t(x) - xe(y) + t(y) = \int_x^y \big(e(y) - e(x')\big)dx'.$$

The integrand, and hence the integral, is nonnegative because e is nondecreasing, proving $xe(x) - t(x) \geq xe(y) - t(y)$. The proof when $x > y$ is symmetric. Finally, verify directly that $t(0) = 0$. $\square$

I use Lemma 2 to eliminate the transfer payment from the representation of competitive search equilibrium:

$$V = \max_{\theta, e} \mu(\theta) \int_0^{\bar{x}} \left(xe(x) - \int_0^x e(y)dy \right) dF(x)$$

$$\text{subject to } c = \frac{\mu(\theta)}{\theta} \int_0^{\bar{x}} \left(\int_0^x e(y)dy \right) dF(x)$$

$$0 \leq e(x) \leq e(y) \leq 1 \text{ for all } x < y.$$

Further simplify using integration-by-parts, $\int_0^{\bar{x}} \left(\int_0^x e(y)dy\right) dF(x) = \int_0^{\bar{x}} e(x)(1-F(x))dx$:

$$V = \max_{\theta,e} \mu(\theta) \left(\int_0^{\bar{x}} xe(x)dF(x) - \int_0^{\bar{x}} e(x)(1-F(x))dx \right) \tag{5}$$

$$\text{subject to } c = \frac{\mu(\theta)}{\theta} \int_0^{\bar{x}} e(x)(1-F(x))dx \tag{6}$$

$$0 \leq e(x) \leq e(y) \leq 1 \text{ for all } x < y \tag{7}$$

That is, a tuple $\{\theta, t, e\}$ is a competitive search equilibrium with asymmetric information if and only if $\{\theta, e\}$ maximizes (5) subject to (6) and (7) and $t(x) = xe(x) - \int_0^x e(y)dy$ for all x. This is a much simpler representation of equilibrium, and is relatively easily manipulated.

3.3 The Lagrangian

Temporarily ignore the monotonicity constraint (7). The problem of maximizing (5) subject to (6) can be represented as a Lagrangian with multiplier $\psi\theta > 0$ on the zero profit condition:

$$\mathcal{L}(\theta, e, \psi) = \mu(\theta) \left(\int_0^{\bar{x}} xe(x)dF(x) + (\psi - 1) \int_0^{\bar{x}} e(x)(1-F(x))dx \right) - \psi\theta c.$$

One may equivalently view this as maximizing a weighted sum of a worker's expected utility $\mu(\theta) \left(\int_0^{\bar{x}} xe(x)dF(x) - \int_0^{\bar{x}} e(x)(1-F(x))dx\right)$ and firm's expected profit $\frac{\mu(\theta)}{\theta} \int_0^{\bar{x}} e(x)(1-F(x))dx - c$, with Pareto weights 1 and $\psi\theta$. By varying ψ, one traces out the Pareto frontier. Competitive search equilibrium picks out a particular point on the Pareto frontier, with firms' expected profit equal to zero.

To solve the Lagrangian, define

$$\phi(x) \equiv x(1-F(x)) + \psi \int_x^{\bar{x}} (y-x)dF(y). \tag{8}$$

The Lagrangian may be expressed as

$$\mathcal{L}(\theta, e, \psi) = -\mu(\theta) \int_0^{\bar{x}} e(x)d\phi(x) - \psi\theta c.$$

Note that I may assume without loss of generality that $e(0) = 0$, since this relaxes the monotonicity constraint on the employment probability function without affecting the value of the objective function. Then using integration-by-parts on the previous expression gives

$$\mathcal{L}(\theta, e, \psi) = \mu(\theta) \int_0^{\bar{x}} \phi(x) de(x) - \psi\theta c. \tag{9}$$

A competitive search equilibrium with asymmetric information is simply a v-u ratio θ and a nondecreasing employment probability function $e(x)$ that maximizes this function, with the transfers determined via $t(x) = xe(x) - \int_0^x e(y)dy$ and the multiplier ψ set to ensure firms earn zero profits.

A cursory examination of (9) reveals some key properties of a competitive search equilibrium with asymmetric information. First, monotonicity of $e(x)$ implies $de(x) \geq 0$ only if $x \in \arg\max_y \phi(y)$. Second, $e(\bar{x}) = 1$ if $\max_y \phi(y) > 0$. And third, assuming $\max_y \phi(y) > 0$, the v-u ratio θ is chosen to maximize $\mu(\theta) \max_x \phi(x) - \psi\theta c$.[4] A characterization of ϕ is therefore critical to a characterization of competitive search equilibrium.

3.4 Threshold Solution to the Employment Probability Function

Under a standard regularity condition, the characterization of the employment probability function is particularly easy because the critical function $\phi(x)$ is single-peaked for any value of ψ. Matches are formed if and only if productivity exceeds an endogenous threshold x^*.

Proposition 4. *Assume F is differentiable and $h(x) \equiv \frac{xF'(x)}{1-F(x)}$ is a nondecreasing function. In a competitive search equilibrium with asymmetric information, the v-u ratio θ and employment threshold x^* satisfy*

$$V = \max_{\theta, x^* \in [0,\bar{x}]} \mu(\theta) x^* (1 - F(x^*)) \tag{10}$$

$$\text{subject to } c = \frac{\mu(\theta)}{\theta} \int_{x^*}^{\bar{x}} (x - x^*) dF(x) \tag{11}$$

and the employment probability function $e(x)$ and transfer function $t(x)$ satisfy

$$e(x) = \begin{cases} 1 \\ 0 \end{cases} \quad \textit{and} \quad t(x) = \begin{cases} x^* & \textit{if} \quad x \in (x^*, \bar{x}) \\ 0 & \quad\quad x \in (0, x^*) \end{cases}. \tag{12}$$

Proof. For any value of ψ, define x^* such that $h(x^*) = 1 - \psi$. (If $h(x) > 1 - \psi$ for all x, set

[4] If $\max_y \phi(y) \leq 0$, θ is chosen to maximize $-\psi\theta c$, and so is either zero or infinite depending on whether ψ is positive or negative.

$x^* = 0$, and if $h(x) < 1-\psi$ for all x, set $x^* = \bar{x}$.) Observe that $\phi(x)$ is increasing when $x < x^*$, $(1-\psi)(1-F(x)) - xF'(x) > 0$, and decreasing when $x > x^*$. Therefore $\phi(x)$ has a unique maximum, x^*, so $e(x) = 0$ when $x < x^*$ and $e(x) = 1$ when $x > x^*$. The value of $t(x)$ follows immediately from Lemma 2, while the representation of the threshold (10)–(11) comes from simplifying the original optimization problem (5)–(7) using the threshold characterization. In particular, a matched worker earns x^* if productivity exceeds this threshold and the firm earns the residual profit $x - x^*$ in the same set of events. $\square$

The condition that $h(x)$ is nondecreasing is related to some more familiar conditions. First, the condition is equivalent to the requirement that the elasticity of $1-F(x)$ with respect to x must be decreasing. A stronger but more familiar requirement is the monotone hazard rate condition that $\frac{F'(x)}{1-F(x)}$ is nondecreasing. This last condition is identical to the requirement that $1-F(x)$ is log-concave. It is satisfied by a broad class of standard distributions, including any distribution with a nondecreasing density p (e.g. the uniform), the normal distribution truncated at zero, the log normal distribution, and the exponential distribution.

Although one typically cannot proceed further than the characterization of competitive search equilibrium in equations (10)–(11), solving for θ and x^* is easy with particular functional forms. An important special case is $\mu(\theta) = \mu_0\theta^{1-\alpha}$, with μ_0 is sufficiently small so that $\mu(\theta) < \min\{\theta, 1\}$ in the relevant parameter range. Then one can eliminate θ from the objective function using the constraint and show that the employment threshold $x^* = \arg\max_x G(x)$, where

$$G(x) \equiv \alpha \log\big(x(1-F(x))\big) + (1-\alpha)\log\left(\int_x^{\bar{x}} (y-x)dF(y)\right).$$

This is a geometric weighted average of the surplus that a worker gets from a meeting, the reservation productivity level when output exceeds that level, $x(1-F(x))$ and the surplus that a firm gets from a meeting, output y in excess of the reservation productivity level x, $\int_x^{\bar{x}}(y-x)dF(y)$. The weights correspond to the elasticity of the total matching rate $\mu_0 u^\alpha v^{1-\alpha}$ with respect to unemployment and vacancies, respectively.

$G(x)$ is a single-peaked function when $h(x)$ is nondecreasing. Observe first that $H(x) \equiv 1 + \frac{x(1-F(x))}{\int_x^{\bar{x}}(1-F(y))dy}$ inherits the monotonicity of $h(x) \equiv \frac{xF'(x)}{1-F(x)}$. To prove this, show by differentiation that $H'(x) \gtreqless 0$ if and only if $H(x) \gtreqless h(x)$. Now suppose $H(x) < h(x)$ for some $x < \bar{x}$. Then $H'(x) < 0$ and $h(x) \geq 0$, so $H(x') < h(x')$ and $H'(x') < 0$ for all $x' > x$. But using L'Hôpital's rule, $\lim_{x\to\bar{x}} H(x)/h(x) = 1$, a contradiction. Therefore $H(x) \geq h(x)$ and

$H'(x) \geq 0$ for all $x < \bar{x}$. Next, differentiate $G(x)$ to show $xG'(x) = 1-(1-\alpha)H(x)-\alpha h(x)$, a non-increasing function, which implies G is single-peaked. After finding the peak of the peak of G, θ is easily computed using the zero profit condition (11).

3.5 General Solution to the Employment Probability Function

Even when $h(x)$ is not monotonic, it is possible to provide a precise characterization of the equilibrium.

Proposition 5. *Assume the distribution function $F(x)$ is generic. In a competitive search equilibrium with asymmetric information, the employment probability function $e(x)$ is a step function with at most two discontinuities. If the employment probability function $e(x)$ has one discontinuity, the v-u ratio θ and threshold x^* solve (10)–(11) and the employment probability function $e(x)$ and transfer function $t(x)$ satisfy (12).*

If there are two discontinuities and F is continuously differentiable, the thresholds x_1^* and x_2^* solve

$$\frac{x_1^* F'(x_1^*)}{1-F(x_1^*)} = \frac{x_2^* F'(x_2^*)}{1-F(x_2^*)} = \frac{\int_{x_1^*}^{x_2^*} x dF(x)}{\int_{x_1^*}^{x_2^*} (1-F(x))dx}, \tag{13}$$

the v-u ratio θ and employment probability e^* solve

$$V = \max_{\theta, e^* \in [0,1]} \mu(\theta)\Big(e^* x_1^*\big(1-F(x_1^*)\big) + (1-e^*)x_2^*\big(1-F(x_2^*)\big)\Big) \tag{14}$$

$$\text{subject to } \theta c = \mu(\theta)\left(e^* \int_{x_1^*}^{\bar{x}} (x-x_1^*)dF(x) + (1-e^*)\int_{x_2^*}^{\bar{x}} (x-x_2^*)dF(x)\right) \tag{15}$$

and the employment probability function $e(x)$ and transfer function $t(x)$ satisfy

$$e(x) = \begin{cases} 1 \\ e^* \\ 0 \end{cases} \quad \text{and} \quad t(x) = \begin{cases} e^* x_1^* + (1-e^*)x_2^* & \\ e^* x_1^* & \text{if} \\ 0 & \end{cases} \begin{matrix} x \in (x_2^*, \bar{x}) \\ x \in (x_1^*, x_2^*) \\ x \in (0, x_1^*) \end{matrix} .$$

Proof. The conclusion that generically there are at most two discontinuities follows from the characterization of the Lagrangian (9). In particular, $de(x) > 0$ only if $x \in \arg\max_y \phi(y)$, where ϕ is defined in (8). There is only a single endogenous variable, the multiplier ψ, in the definition of ϕ, so for generic functions F and arbitrary (not necessarily generic) values of ψ, the function ϕ has at most two maxima. The characterization of the case of a single discontinuity follows the proof of Proposition 4.

Next suppose there are two discontinuities and F is continuously differentiable. At any $x^* \in \arg\max \phi(x)$, the first order conditions indicate that $\phi'(x^*) = 0$, i.e. $x^*F'(x^*) = (1-\psi)(1-F(x^*))$. Moreover, if $\arg\max \phi(x)$ is not a singleton, it contains two points $x_1^* < x_2^*$ with

$$\begin{aligned} 0 &= \phi(x_2^*) - \phi(x_1^*) \\ &= x_2^*(1-F(x_2^*)) - x_1^*(1-F(x_1^*)) - \psi \int_{x_1^*}^{x_2^*} (1-F(x))dx \\ &= -\int_{x_1^*}^{x_2^*} x dF(x) + (1-\psi) \int_{x_1^*}^{x_2^*} (1-F(x))dx, \end{aligned}$$

where the second equality uses the definition of ϕ and the third uses integration-by-parts. This gives three conditions for x_1^*, x_2^*, and ψ, which can be written as

$$1-\psi = \frac{x_1^*F'(x_1^*)}{1-F(x_1^*)} = \frac{x_2^*F'(x_2^*)}{1-F(x_2^*)} = \frac{\int_{x_1^*}^{x_2^*} x dF(x)}{\int_{x_1^*}^{x_2^*} (1-F(x))dx}.$$

Finally, the transfer function $t(x)$ is determined from Lemma 2. The problem describing the choice of θ and e^* (conditional on x_1^* and x_2^*) comes from simplifying the original optimization problem (5)–(7) using the characterization given above. □

When F is not continuously differentiable, the definition of x_1^* and x_2^* must be appropriately generalized. Of course, even in the case where F is continuously differentiable, it is possible that multiple pairs of points satisfy the restriction (13), in which case any such pairs are potentially part of an equilibrium.

It is worth noting a simple way of implementing this contract. A firm commits that when it contacts a worker, it will randomly set a wage, equal to x_1^* with probability e^* and x_2^* with probability $1-e^*$. The firm also reserves the right to decide whether to hire the worker at the chosen wage. If realized productivity $x < x_1^*$, the worker is never hired and is paid nothing. If $x \in (x_1^*, x_2^*)$, the worker is hired if the firm chose the low wage, in which event it is paid x_1^*, giving an employment probability of e^* and an expected transfer of $e^*x_1^*$. Finally, if $x > x_2^*$, the worker is hired in either event, giving an expected transfer of $e^*x_1^* + (1-e^*)x_2^*$. This recollects the equilibrium of Curtis and Wright (2003), in which a fraction e^* firms pay the low wage x_1^* and the remaining firms pay the high wage x_2^*; in the competitive search model, each firm randomly selects the wage.

Once again, it is relatively straightforward to solve for θ and e^* when $\mu(\theta) = \mu_0\theta^{1-\alpha}$. Use the zero profit condition to eliminate θ from the objective function, yielding an unconstrained optimization problem for the choice of the employment probability in the intermediate range: e^* maximizes

$$\alpha \log \Big(e^* x_1^* \big(1 - F(x_1^*)\big) + (1-e^*) x_2^* \big(1 - F(x_2^*)\big)\Big) \\ + (1-\alpha) \log \left(e^* \int_{x_1^*}^{\bar{x}} (x - x_1^*) dF(x) + (1-e^*) \int_{x_2^*}^{\bar{x}} (x - x_2^*) dF(x) \right).$$

This again can be interpreted as a geometric average of the workers' and firms' surplus from matching. It is possible to solve recursively first for e^* and then for the v-u ratio θ from the zero profit condition (15). If this problem admits a solution with $e^* \in (0,1)$, one must then compare the value using public randomization—maximization of (14) subject to (15)—to the value obtained without public randomization—maximization of (10) subject to (11).

3.6 An Example with Public Randomization

I verify by example that mixed strategies may be used in a competitive search equilibrium with asymmetric information even when the type distribution is atomless. Consider the density function $F'(x) = 0.1 + 3x(x-1)^2(2-x)$ with support $[0,2]$, a quartic equation with local minima at $x = 0$, 1, and 2 and maxima at $1 \pm 1/\sqrt{2}$. The critical function $h(x) = \frac{xF'(x)}{1-F(x)}$ is increasing for $x < 0.550$ and $x > 0.979$ but is decreasing in the intermediate interval. This makes it possible to find values of x_1^* and x_2^* satisfying the required pair of equations, $x_1^* = 0.349$ and $x_2^* = 1.142$. I therefore look for an equilibrium in which $e(x) = e^* \in (0,1)$ when x lies in this intermediate region.

To proceed further, I must assume a functional form for the matching function, $\mu(\theta) = \mu_0\theta^{1/2}$. Then maximizing (14) subject to (15) indicates that $e^* = 0.723$, regardless of the cost of a vacancy and the matching function constant, yielding the worker value $0.194\mu_0^2/c$.[5] Alternatively, if the firm considers not using a public randomization device, it may instead choose a single employment threshold x^*. Then regardless of c and μ_0, the (constrained) optimal threshold is $x^* = 0.457$, yielding value $0.190\mu_0^2/c$, 2.1 percent less than that feasible with public randomization. I conclude that there is public randomization in any competitive

[5] In addition, $\theta = 0.311\mu_0^2/c^2$, so $\mu(\theta) < \min\{\theta, 1\}$ if $\mu_0^2 < 1.792c < 1$. With symmetric information, the v-u ratio is lower, $\theta = 0.25\mu_0^2/c^2$, and the worker's value is higher, $0.25\mu_0^2/c$.

search equilibrium with asymmetric information with these density and matching functions.

3.7 The Effect of Asymmetric Information

This paper has thus far provided two examples, one with two types of jobs and one with a continuum of jobs. In both, asymmetric information weakly increases the v-u ratio compared to the symmetric information benchmark. This section proves that this result holds more generally.

Proposition 6. *Let θ^a denote the v-u ratio in a competitive search equilibrium with asymmetric information and θ^s denote the v-u ratio in a competitive search equilibrium with symmetric information. Then $\theta^a > \theta^s$.*

Proof. Equation (4) implies that $\mu'(\theta^s)\int_0^{\bar{x}} xdF(x) = c$, while the Lagrangian (9) implies θ^a satisfies

$$\mu'(\theta^a)\max_x \phi(x) = \psi c.$$

Since for any fixed ψ, $\phi(x) > 0$ at x sufficiently near $\bar{x}$, the left hand side is positive, which implies $\psi > 0$. Then since μ is concave, the result follows if $\max_x \phi(x) > \psi \int_0^{\bar{x}} xdF(x)$. But equation (8) implies $\phi(0) = \psi \int_0^{\bar{x}} xdF(x)$. Moreover, $0 \in \arg\max \phi(0)$ only if the competitive search equilibrium with symmetric information satisfies the information constraints, a possibility that I have already precluded. □

To provide some intuition for this result, note that setting $\theta = \theta^s$ and $e(x) = 1$ for all x violates the zero profit constraint (6):

$$\frac{\mu(\theta^s)}{\theta^s}\int_0^{\bar{x}}(1-F(x))dx = \frac{\mu(\theta^s)}{\theta^s}\int_0^{\bar{x}} xdF(x) > c,$$

since concavity of μ implies $\frac{\mu(\theta)}{\theta} > \mu'(\theta)$. Restoring zero profits therefore requires either reducing $e(x)$ for some x or raising θ so as to lower the rate that firms contact workers. Optimality dictates undertaking both actions.

4 Dynamic Model

This section develops a dynamic extension to the model in the continuous type model. Time is continuous and all agents are infinitely lived and discount future income at rate r. At any point in time, a firm may open a vacancy and advertise an employment contract at a flow cost c. Unemployed workers observe all the advertised contracts and direct their search accordingly. I assume that the flow matching rate $\mu(\theta)$ is increasing and concave with $\mu(0) = 0$. When a worker and firm meet, the firm observes the match specific productivity realization but the worker does not. Match productivity is constant, but the job ends exogenously at rate s.

As a preliminary step, note that a risk-neutral worker and firm care only about the expected present value of transfers conditional on a particular productivity announcement. I therefore assume without loss of generality that an employment contract has the following form: the firm announces the productivity realization x, hires the worker with probability $e(x)$, gives the worker a lump-sum transfer $t(x)$, and thereafter pays the worker her reservation wage rV while the worker is employed, where V is the expected present value of income for an unemployed worker. With this structure, it is possible to represent a competitive search equilibrium with asymmetric information as

$$
\begin{aligned}
rV = \max_{\theta,t,e}\ & \mu(\theta) \int_0^{\bar{x}} t(x) dF(x) \\
\text{subject to } c = & \frac{\mu(\theta)}{\theta} \int_0^{\bar{x}} \left(e(x) \frac{x - rV}{r+s} - t(x) \right) dF(x) \\
& e(x) \frac{x - rV}{r+s} - t(x) \geq e(y) \frac{x - rV}{r+s} - t(y) \text{ for all } \{x, y\} \in [0, \bar{x}]^2 \\
& t(0) = 0 \\
& e(x) \in [0, 1] \text{ for all } x \in [0, \bar{x}]
\end{aligned}
$$

The worker chooses the v-u ratio, lump-sum transfers, and employment probabilities in order to maximize her expected utility, where her flow utility is the rate at which she contacts a firm $\mu(\theta)$ times the expected capital gain $\int_0^{\bar{x}} t(x)dF(x)$. The firm must earn zero profits from its vacancy, so the flow cost of a vacancy c equals the product of the rate at which a firm contacts a worker $\mu(\theta)/\theta$ and the expected capital gain: if the firm truthfully announces that productivity is x, it hires the worker with probability $e(x)$ giving present value $\frac{x-rV}{r+s}$, but it must pay the transfer $t(x)$. Finally, the worker's choice must satisfy the incentive constraint, so a firm prefers to truthfully announce that productivity is x rather than report y, and it must not insist on any transfers when the firm announces that productivity is zero.

With symmetric information, one can again eliminate the transfer function using the zero profit constraint to get

$$rV = \max_{\theta,e} \mu(\theta) \int_0^{\bar{x}} e(x) \frac{x - rV}{r+s} dF(x) - \theta c.$$

Clearly $e(x) = 1$ if $x > rV$ and 0 otherwise. Equivalently, the v-u ratio θ and employment threshold and flow value of unemployment $x^* = rV$ must satisfy

$$x^* = \max_{\theta} \frac{\mu(\theta)}{r+s} \int_{x^*}^{\bar{x}} (x - x^*) dF(x) - \theta c.$$

But such an allocation will generally violate the incentive constraints. With asymmetric information, it is possible to replicate the proof of Lemma 2 to show that

Lemma 3. *The following two conditions are identical:*

$$\begin{aligned} &A: \quad e(x)\frac{x - rV}{r+s} - t(x) \geq e(y)\frac{x - rV}{r+s} - t(y) \text{ for all } \{x, y\} \in [0, \bar{x}]^2 \text{ and } t(0) = 0 \\ &B: \quad t(x) = \frac{e(x)(x - rV) - \int_0^x e(y)dy}{r+s} \text{ and } e(x) \text{ is a nondecreasing function.} \end{aligned}$$

Substituting this into the constrained optimization problem and using the standard integration-by-parts tricks gives

$$\begin{aligned} (r+s)rV = \max_{\theta,e} \; &\mu(\theta) \left(\int_0^{\bar{x}} (x - rV)e(x)dF(x) - \int_0^{\bar{x}} e(x)(1 - F(x))dx \right) \\ \text{subject to } &\theta(r+s)c = \mu(\theta) \int_0^{\bar{x}} e(x)(1 - F(x))dx \\ &0 \leq e(x) \leq e(y) \leq 1 \text{ for all } 0 \leq x \leq y \leq \bar{x} \end{aligned}$$

In particular, if we ignore the monotonicity constraint, we may represent this as a Lagrangian by placing the multiplier ψ on the constraint:

$$\mathcal{L}(\theta, e, \psi) = \mu(\theta) \left(\int_0^{\bar{x}} (x - rV)e(x)dF(x) + (\psi - 1) \int_0^{\bar{x}} e(x)(1 - F(x))dx \right) - \psi\theta(r+s)c$$

Now assume F is differentiable. If $h(x) = \frac{xF'(x)}{1-F(x)}$ is nondecreasing then $\frac{(x-rV)F'(x)}{1-F(x)}$ is nondecreasing in x as well for any $x \geq rV \geq 0$. This implies there is an $x^* \geq rV$ such that $(x-rV)F'(x) \leq (1-\psi)(1-F(x))$ if $x < x^*$ and $(x-rV)F'(x) \geq (1-\psi)(1-F(x))$ if $x > x^*$. Optimality then requires using an employment threshold of x^*. Substituting the threshold rule back into the previous problem and solving the objective function for V implies that a competitive search equilibrium with asymmetric information is a tuple $\{V, \theta, x^*\}$ solving

$$rV = \max_{\theta, x^*} \frac{\mu(\theta)x^*(1 - F(x^*))}{r + s + \mu(\theta)(1 - F(x^*))}$$
$$\text{subject to } c = \frac{\mu(\theta)}{\theta(r + s)} \int_{x^*}^{\bar{x}} (x - x^*) dF(x)$$

a simple static optimization problem. It is easy to verify that $x^* > rV$,[6] so asymmetric information forces the worker and firm to waste some potential gains from trade. The case when $h(x)$ is not monotone is similarly unaffected.

An interesting issue that arises in the dynamic model is an alternative interpretation of the employment probability $e(x)$. All that matters is that the firm only enjoy a fraction $e(x)$ of the potential gains from trade $(x - rV)/(r + s)$. This can be achieved through probabilistic hiring, but it can also be achieved through a premature layoff. That is, if a firm commits to hire a worker for at most T periods, the match surplus falls to

$$\int_0^T e^{-(r+s)\tau}(x - rV)d\tau = \frac{x - rV}{r + s}\left(1 - e^{-(r+s)T}\right).$$

Then by setting $T(x) = \frac{-\log(1 - e(x))}{r + s}$, the contract achieves the desired outcome in a deterministic and verifiable fashion.

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[6] If $x^* = rV$, the objective function implies $V = 0$ as well. But it is easy to find values of θ and x^* consistent with $V > 0$, a contradiction.

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